Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

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10MAT41

Fourth Semester B.E. Degree Examination, June/July 2015

Engineering Mathematics - IV

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART - A

1 a. Obtain y(0.2) using Picards method upto second iteration for the initial value problem $\frac{dy}{dx} = x^2 - 2y \quad y(0) = 1$. **(06 Marks)**

b. Solve by Eulers modified method to obtain y(1.2) given $y' = \frac{y+x}{y-x}$ $y(1) = 2$. **(07 Marks)**

c. Using Adam Bash forth method obtain y at x = 0.8 given $\frac{dy}{dx} = x - y^2$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$ and $y(0.6) = 0.1762$. **(07 Marks)**

2 a. Solve by 4^{th} order Runge Kutta method simultaneous equations given by $\frac{dx}{dt} = y - t$, $\frac{dy}{dt} = x + t$ with $x = 1 = y$ at $t = 0$, obtain y(0.1) and x(0.1). **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$, $y(0) = 1$, $y'(0) = 0$. Evaluate y(0.2) correct to four decimal places, using Runge Kutta method of fourth order. **(07 Marks)**

c. Solve for x = 0.4 using Milnes predictor corrector formula for the differential equation $y'' + xy' + y = 0$ with $y(0) = 1$, $y(0.1) = 0.995$, $y(0.2) = 0.9802$ and $y(0.3) = 0.956$. Also $z(0) = 0$, $z(0.1) = -0.0995$, $z(0.2) = -0.196$, $z(0.3) = -0.2863$. **(07 Marks)**

3 a. Verify whether f(z) = sin2z is analytic, hence obtain the derivative. **(06 Marks)**

b. Determine the analytic function f(z) whose imaginary part is $\frac{y}{x^2 + y^2}$. **(07 Marks)**

c. Define a harmonic function. Prove that real and imaginary parts of an analytic function are harmonic. **(07 Marks)**

4 a. Under the mapping $w = e^z$, find the image of i) $1 \le x \le 2$ ii) $\pi/3 < y < \frac{\pi}{2}$. **(06 Marks)**

b. Find the bilinear transformation which maps the points 1, i, -1 from z plane to 2, i, -2 into w plane. Also find the fixed points. **(07 Marks)**

c. State and prove Cauchy's integral formula. **(07 Marks)**

PART - B

5 a. Prove $J_n(x) = \frac{x}{2n} [J_{n-1}(x) + J_{n+1}(x)]$. **(06 Marks)**

b. Prove $(n+1) P_n(x) = (2n+1) x P_n(x) - n P_{n-1}(x)$. **(07 Marks)**

c. Explain the following in terms of Legendres polynomials.
$x^4 + 3x^3 - x^2 + 5x - 2$ **(07 Marks)**

6 a. A class has 10 boys and 6 girls. Three students are selected at random one after another. Find the probability that i) first and third are boys, second a girl ii) first and second are of same sex and third is of opposite sex. **(06 Marks)**

b. If $P(A) = 0.4$, $P(B/A) = 0.9$, $P(\overline{B}/\overline{A}) = 0.6$. Find $P(A/B)$, $P(A/\overline{B})$. **(07 Marks)**

c. In a bolt factory machines A, B and C manufacture 20%, 35% and 45% of the total of their outputs 5%, 4% and 2% are defective. A bolt is drawn at random found to be defective. What is the probability that it is from machine B? **(07 Marks)**

7 a. A random variable x has the following distribution :

x :	-2	-1	0	1	2	3	4
P(x) :	0.1	0.1	k	0.1	2k	k	k

Find k, mean and S.D of the distribution. **(06 Marks)**

b. The probability that a bomb dropped hits the target is 0.2. Find the probability that out of 6 bombs dropped i) exactly 2 will hit the target ii) atleast 3 will hit the target. **(07 Marks)**

c. Find the mean and variance of the exponential distribution. **(07 Marks)**

8 a. A die is tossed 960 times and 5 appear 184 times. Is the die biased? **(06 Marks)**

b. Nine items have values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from assumed of mean of 47.5. ($\gamma = 8$, $t_{0.05} = 2.31$). **(07 Marks)**

c. A set of 5 similar coins tossed 320 times gives following table.

No. of heads :	0	1	2	3	4	5
Freq.	6	27	72	112	71	32

Test the hypothesis that data follows binomial distribution (Given $\gamma = 5$, $\chi^2_{0.05} = 11.07$) **(07 Marks)**

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